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ABSTRACT

The purpose of this report was to test the hypothesis that extra-scope transfer depends on the extent to which a statement of strategy may be viewed as a restriction of a more general strategy. Sixty-six high school students were taught a restricted statement of one of three strategies of varying generality. Twenty-two of these students served as a control group. All subjects were tested on six problems which were based on a variant of the game "NIM". The first two problems were within the scope of the most specific strategy, the second two were within the scope of more general strategies, and the last two were within the scope of the most general strategy. Results indicate that restricted rule statements may provide an adequate basis for generalization. Also, performance on the second problem of each pair was contingent on performance on the corresponding first problems indicating that what is learned may be determined by performance on single test items and used to predict performance on additional similar-scope problems. Suggestions are made for future research. (Author/FL)

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EXTRA-SCOPE TRANSFER IN LEARNING MATHEMATICAL RULES

Joseph M. Scandura
University of Pennsylvania

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
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Joseph M. Scandura
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Summary

The notion of a restricted rule or strategy was introduced. It was hypothesized that extra-scope transfer depends on the extent to which a statement of strategy may be viewed as a restriction of a more general strategy. Sixty six high school subjects were taught a restricted statement (S', SG', or G') of one of three strategies of varying generality, S(=S')$\prec$ SG (SG')$\prec$ G (G'). Twenty two subjects served as a control (C). All subjects were tested on six problems, the first two within the scope of the most specific strategy (S), the second two within the scope of only the more general strategies (SG and G), and the last two only within the scope of strategy G. Statements S', SG', and G' were directly applicable only to the first two problems. Groups SG' and G' evidenced extra-scope transfer. Groups S' and C did not. In additon, performance on the second problem of each pair was contingent on performance on the corresponding first problems indicating that "what is learned" may be determined by performance on single test items and used to predict performance on additional similar-scope problems. Suggestions were made for future research.

Introduction

Scandura, Woodward, and Lee (1967) demonstrated that performance on transfer tasks is generally in accord with the logically determined scope of rule and strategy statements.* In each of two experiments, Ss were presented with one of three statements of rules (or strategies) of varying generality and were tested on three problems. The first problem was within the scope of all three rules; the second, within the scope of only the two more general rules; and the third, within the scope of only the most general rule. In most instances, there was essentially no difference in the level of performance on the within-scope problems and no extra-scope transfer (to problems to which the rule did not directly apply).

There was one glaring exception involving extra-scope transfer. In experiment II, Ss given the statement, "50 x 50," which was directly applicable only to Problem 1, performed equally as well on Extra-Scope Problem 2 as did those Ss given the statement, "n x n," where the dimension (i.e., variable) n was allowed to vary over the positive integers. This result obtained even though "n x n" was directly applicable to both Problems 1 and 2. While the study itself was inadequate to specify the source of this transfer, a post hoc analysis of the experimental treatments indicated that "50 x 50" was the only rule statement included in the study which was in some sense a restriction of a more general rule or strategy. The statement, "50 x 50," could be obtained from the general statement "n x n," by replacing the response determining dimension, n, by the value 50. More generally it would appear that a restricted statement may be viewed as one obtained by replacing response-determining dimensions (see Scandura, 1966, 1967 , 1968b) in the statement of a general rule or strategy with the specific values of a particular instance. The authors, therefore, conjectured that a restricted rule statement might well provide a basis for generalization to all problems within the scope of the corresponding unrestricted rule. The primary purpose of this study was to test this hypothesis.

A secondary purpose was to obtain further information on the "consistency" hypothesis. Under certain conditions, it has been found that transfer to one instance of a rule almost invariably implies transfer to other instances of the rule (Scandura, 1966, 1967, 1969a: Scandura et al., 1967). As was the case with extra-scope transfer, however, one exception to the consistency hypothesis was obtained in the study by Scandura et al. (1967). The level of performance on one within-scope problem was considerably below that on the others. Whereas the response determining values of the homogeneous problems differed along a single dimension, the exceptional within-scope problem differed along a second dimension as well. Taking this observation into account, a modified form of the consistency hypothesis was advanced. It was hypothesized that if transfer to one problem indicates that a particular rule or strategy (e.g., "50 x 50") has been generalized along one or more familiar dimensions (e.g., to "n x n") then transfer to additional problems along the same dimensions (and within the scope of a less restrictive rule) should also be expected.

Method

Material

The material was based on a variant of the number game, "NIM." In this

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*The terms "rule" and "strategy" are used synonymously throughout this paper. While "rule" is the preferred technical term (e.g., Scandura, 1968a), "strategy" better connotes more complex multiphased rules of the sort used in this study.

variant, two players alternately select numbers from a specified set of consecutive integers (including 1) and keep a running sum. The winner is the one who picks the last number in a series having a predetermined sum. Each such game can be characterized by an ordered pair (n, m) where the corresponding value of n is the largest integer in the selection set and the value of m is the predetermined sum (n and m refer to dimensions over which NIM may vary). If the set consist of the integers 1-6 and the sum is 31, the players alternately select numbers 1-6 until the cumulative sum is either 31 or above (in which case no one wins).

Scandura et al. (1967) presented statements of three general rules by which the person making the first selection can always win. The specific (S) rule is sufficient for winning only (6, 31) games and was stated:

> In order to win the game you should make 3 your first selection. Then you should make selections so that the sums corresponding to your selections differ by 7.

The specific-general (SG) rule, an unidimensional strategy, is applicable to any game of the form (6, m) and was stated:

> In order to win the game, the appropriate first selection is determined by dividing the desired sum by 7. The remainder of this division is precisely the selection which should be made first. . . .

The general (G) rule, a bidimensional strategy, is applicable to any (n, m) game, where both n and m are allowed to vary, and was stated:

> In order to win the game the appropriate first selection is determined by adding one to the largest number in the set from which the selections must come and dividing the desired sum by this result. The remainder of this division is precisely the selection that should be made first. Then you should make selections so that the sums corresponding to your selections differ by one greater than the largest number in the set from which the selections must come.

Statements of restrictions of these strategies, which are applicable only to (6, 31) games, were constructed for use in this experiment. Rule S' was essentially identical to rule S and was stated in the same way. Rule SG' was a restriction of unidimensional strategy, SG, in the sense that SG was restricted to one value (i.e., 31) of the desired-sum (m) dimension. Rule SG' was stated:

> The appropriate first selection is determined by dividing 31 by 7. The remainder 3 should be your first selection. . . .

Rule G' was a restriction of bidimensional strategy, G, in that G was restricted to one value (i.e., 31 and 6, respectively) of the desired-sum (m) and size-of-selection-set (n) dimensions. Rule G' was stated:

> The appropriate first selection is determined by adding one to six, (1 + 6), and dividing 31 by this result. The remainder 3 of this division is the selection which should be made first. . . . It is important to notice that 7 = 6 + 1.

The materials were reproduced by mimeograph and were combined into nine different 8-1/2 x 11 inch booklets--an introduction, four treatments, and four tests. The introduction consisted of 4 pages and was designed to insure that Ss knew the objective of and how to play NIM. Page 1 indicated that the experimental results would be made available to Ss, asked that they not divulge information about the experiment to others who might participate, and explained the nature of the (6, 31) game. Page 2 consisted of one completed (6, 31) example and a (6, 31) practice game in which S was required to compute the running sums in accordance with a specified sequence of selections. Knowledge of results was given on page 3 along with another (6, 31) practice game with the result of the latter given on page 4. Nothing was said in the introduction about game-winning rules, but it was mentioned that there are many variations of NIM.

Three of the four treatment booklets included one of the restricted statements (S', SG', and G') on page 1 together with a common (6, 31) game which was provided for practice. The solution to this (6, 31) game was given on page 2 and Ss were instructed to replay the same game, on page 3 after correcting any previous errors. In this common (6, 31) game, the running sums were 3, 5, 10, 11, 17, 20, 24, 25, 31. The fourth booklet served as a control. It consisted solely of the common (6, 31) example with no statement of a game-winning rule. Nonetheless, by remembering those sums which corresponded to the winning selections (i.e., 3, 10, 17, 24, 31), an S might conceivably win any new (6, 31) game.

The four test booklets corresponded to the four treatment booklets. Page 1 was common to all test booklets and explained how to use the booklet. Each of the successive pages (2-7) included one common test game together with that game-winning statement (S', SG', or G') associated with the corresponding treatment booklet. This procedure was followed to help eliminate errors due to recall. The "opponent's" selections were printed in the booklet and S was instructed to make his selections and to compute the running sums. The first two problems, 1A and 1B, were (6, 31) games. Problems 2A and 2B were (6, m) games which differed along the desired-sum dimension, with m = 25 and m = 29, respectively. Problems 3A and 3B were (n, m) games, which differed along both the desired-sum and size-of-selection-set dimensions, with n = 5, m = 26 and n = 7, m = 33, respectively.

Subjects, Design, and Procedure

The Ss were 88 West Philadelphia High School students enrolled in an academic mathematics program. They were randomly assigned to three experimental groups (S', SG', G') and a control (C) so that each group included 22 Ss.

Each S completed the introductory booklet, one of the four treatment booklets, and the corresponding test booklet, in that order. The S was told to read the material carefully. The experiment was self-paced and with only a few exceptions Ss completed the experiment well within the time limit of 40 minutes.

The criterion measure was use of the appropriate pattern (AP). The S was was given credit for using the AP if he won the game and employed an appropriate

game-winning strategy. All of the tests conducted were applied to 2 x 2 contingency tables. When the measures were independent, the exact Fisher-Yates formula was used (Finney, 1948); when correlated, a different nonparametric test, based on χ^2, was used (McNemar, 1955, pp. 358-359). One-tailed tests were used in conjunction with the stated hypotheses with an alpha level of .05.

Results and Conclusions

Table 1 shows that restricted rule statements may provide an adequate basis for generalization. Statements of unidimensional and bidimensional strategies, even when restricted to particular values of these dimensions, may result in transfer to new problems which differ from the training problem (e.g., common example) along these same dimensions. The three experimental groups performed at essentially the same level on problems 1A and 1B, but there were 12 Ss in Groups SG' and G', as compared to none in Group S', who were successful on problems 2A and 2B. This difference was significant at the .01 level.

TABLE 1

Number of Appropriate Patterns

Group	N	Problem					
		1A (6, 31)	1B (6, 31)	2A (6, 25)	2B (6, 29)	3A (5, 26)	3B (7, 33)
C	22	0	0	0	0	0	0
S'	22	16	16	0	0	0	0
SG'	22	19	20	7	7	2	2
G'	22	18	18	5	5	0	2

Note.--Abreviated: C = control, S' = restricted specific, SG' = restricted specific-general, G' = restricted general.

A cursory review of the literature suggests that the transfer observed in a number of other studies may also have involved generalizing along one or more dimensions of a restricted rule statement. Maier (1945), for example, found that providing S with a problem-solving strategy, as it applied to one problem (i.e., with a restricted statement), improved the level of performance on a second problem (which was presumably within the scope of a more general strategy). Some such generalization mechanism may also be involved in what some investigators have called "remote transfer." Thus, in a recent study, Wittrock's (1967) nonreplace-

ment-strategy group was presented with a restriction of a general strategy which was applicable to his remote transfer items. Apparently, what these Ss actually learned (i.e., discovered) was the more general strategy,*

The performance of the G' Ss, however, suggests that transfer cannot necessarily be expected to all problems within the scope of the rule from which a restricted statement is derived. Of the five Ss in Group G' who were successful on problems 2A and 2B, none was successful on problem 3A and only two, on problem 3B. These differences between problems 2A and 2B and problems 3A and 3B suggest that the level of performance on transfer problems may depend on the particular dimension(s) involved. Problems 2A and 2B required that the G' statement be generalized only along the desired-sum dimension whereas problems 3A and 3B required generalization along the size-of selection-set dimension as well. Apparently, the G' Ss were more capable of making the former generalization than the latter.

The authors also feel obliged** to comment on the fact that two SG' Ss generalized beyond the scope of rule SG to problems 3A and 3B. These SG' Ss were apparently as able to generalize along the size-of-selection-set dimension as were the two G' Ss who were successful on problem 3B. Thus, the statement cue, "7," in statement SG' was equally as helpful as the cue, "6 + 1," in statement G' even though "6" in the latter cue corresponded directly to the number of integers in the selection set. (The former cue, "7," was one larger.) The S' Ss, on the other hand, seemed uniformly unable to generalize along either dimension. To do so, they would have had to have observed that the desired sum, 31, when divided by the constant difference, 7, leaves a remainder of 3 (the first selection).

These observations suggest that the ease with which response-determining properties of an illustrative (training) problem can be related to the corresponding response-determining value (cue) in a restricted statement may have an important effect on the extent of transfer. A pilot study conducted with 20 highly motivated and mathematically oriented doctoral students at the Univerity of Pennsylvania tends to provide further support for this interpretation. All of the SG' and G' Ss and four out of five of the S' Ss were able to generalize to problems 2A, 2B, 3A, and 3B. Clearly, the ease with which a correspondence can be determined between the determining properties of an illustrative problem and statement cues depends on individual differences as well as on the nature of the cue. A major task of future research will be to deter-

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*Many psychologists feel that "what is learned" is excess theoretical baggage since the notion must invariably be defined in terms of transfer. While admitting the ultimate necessity of operational definition, the authors take the position that "what is learned" is a useful construct. In particular, performance on two test items (one training and one transfer) can often be used to identify "what is learned" by individual Ss, thereby making it possible to predict their performance on additional transfer items. This latter assertion is well exemplified by the present consistency data.

**A program of ongoing research by the first author and his collaborators is aimed at uncovering laws of mathematical learning and behavior which hold in a deterministic (or near-deterministic) sense. Thus, when exceptions occur, even where the effects are not "statistically reliable," they are viewed as facts to be explained and not probabilistic deviations which may be safely ignored. Although both the behaviors in question and the methods of approach differ greatly, the authors' research objectives are quite similar to those adopted long ago by Skinner and his followers--to uncover idiographic laws.

mine what the important individual differences are.

To test the consistency hypothesis, those Ss who used the AP on problems 1A, 2A, and 3A and those who did not (non-AP users) were compared as to AP use on problems 1B, 2B, and 3B, respectively. There were significantly more AP users on problem 1A who were AP users on problem 1B than was the case for non- AP users on problem 1A ($p < .001$). The same relationship held for problems 2A and 2B ($p < .001$) and problems 3A and 3B ($p < .001$), respectively. There were only 4 cases out of a total of 131 in which a non-AP user (in Groups S', SG', and G') on an A problem became an AP user on the corresponding B problem. There was only 1 case (out of 67) where an AP user on an A problem was not an AP user on the corresponding B problem.

These results suggest that if transfer obtains on one new problem, which differs (from the training problem) along one or more dimensions, then transfer may be expected to other problems which differ along these same dimensions. Of course, the boundary conditions for this assertion still need to be determined. At the very least, it would seem that the dimension (s) in question would have to be familiar to Ss (but just what this familiarity entails is not entirely clear).

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APPENDIX I

EXPERIMENTAL MATERIALS

NAME_____

GENERAL DIRECTIONS

This experiment is designed to determine how well you can relate and generalize certain mathematical patterns. You will be given some material to learn and then be tested on this material. The results of this test will be made known to your instructor and he may pass this information on to you.

Other people may be participating in this experiment at a later date, so please do not spoil the experiment by talking to anyone about it. Your cooperation is appreciated.

As a participant in this experiment, you are going to be asked to learn to play a game. The game is a number game played between two people. The game has many variations, but at present we will examine only one of the possibilities. In order to learn to play, you must read very carefully.

The game is initiated by one person making a selection of a number from the set {1, 2, 3 4 5, 6}. Participants then make alternating selections from this set and a running sum is kept. A number may be selected more than once. The object of the game is to make the selection which makes the sum exactly 31.

EXAMPLE 1

John is playing against Mary

	Running sum	
John selects 4	4	
Mary selects 2	6	(from 4 + 2)
John selects 5	11	(from 6 + 5)
Mary selects 4	15	(from 4 + 11)
John selects 6	21	(from 6 + 15)
Mary selects 6	27	(from 6 + 21)
John selects 4	31	(from 4 + 27)

John wins since his last selection made the sum 31.

Note that John and Mary make alternating selections.

EXAMPLE 2

You fill in the blanks

	Running sum
John selects 3	3
Mary selects 3	6
John selects 6	___
Mary selects 4	16
John selects 5	___
Mary selects 2	23
John selects 4	___
Mary selects 4	31

_____ wins the game.

Turn to the next page to check your answers.

SOLUTION FOR EXAMPLE 2

	Running sum	
John selects 3	3	
Mary selects 3	6	
John selects 6	12	(from 6 + 6)
Mary selects 4	16	
John selects 5	21	(from 5 + 16)
Mary selects 2	23	
John selects 4	27	(from 4 + 23)
Mary selects 4	31	

Mary wins the game. (Because she made the selection which resulted in the sum of 31.)

After you have completed checking your answers, go on to the next example.

EXAMPLE 3

Fill in the blanks

	Running sum
Mary selects 6	___
John selects 6	___
Mary selects 5	___
John selects 3	___
Mary selects 2	___
John selects 4	___
Mary selects 5	___

_____ wins the game.

Turn to the next page to check your answers.

SOLUTION FOR EXAMPLE 3

	Running sum
Mary selects 6	6
John selects 6	12
Mary selects 5	17
John selects 3	20
Mary selects 2	22
John selects 4	26
Mary selects 5	31

___Mary___ wins the game.

So far we have discussed only the {1, 2, 3, 4, 5, 6} and sum 31 game. As suggested on page 1, the game has many variations. These variations come from varying the allowable selections and also the desired sum. Thus, if we allow selections from the set {1, 2, 3, 4, 5} and allow the desired sum to be 23, we get a game similar to the one described.

CONTROL MATERIALS BOOKLET

There is a procedure which will enable you to win any (6,31) game whenever you are allowed to make the first selection.

Let us try the procedure on Example 4.

	Sum
You go first.	
I select ___.	___
Your opponent selects 2.	___
I select ___.	___
Your opponent selects 1.	___
I select ___.	___
Your opponent selects 3.	___
I select ___.	___
Your opponent selects 1.	___
I select ___ and win.	___

Solution to Example 4 (6,31)

	Sum
You select 3.	3
Your opponent's selection is 2.	5
You select 5.	10
Your opponent's selection is 1.	11
You select 6.	17
Your opponent's selection is 3.	20
You select 4.	24
Your opponent's selection is 1.	25
You select 6 and win.	31

Check your solution to example 4 with the solution given and repeat the example again on the next page.

There is a procedure which will enable you to win any (6,31) game whenever you are allowed to make the first selection.

Let us try the procedure on Example 4.

	Sum
You go first.	
I select ___.	___
Your opponent selects 2.	___
I select ___.	___
Your opponent selects 1.	___
I select ___.	___
Your opponent selects 3.	___
I select ___.	___
Your opponent selects 1.	___
I select ___ and win.	___

Check again with the solution to see if you did example 4 correctly. If not, go back and do example 4 until it is correct.

When you think you understand the procedure for winning the (6, 31) game, turn the page and read the test instructions carefully.

S' TREATMENT MATERIALS

There is a procedure which will enable you to win any (6,31) game whenever you are allowed to make the first selection.

The appropriate first selection should be the integer 3.

Your later choices should be made so that your opponent's preceding choice plus your choice add up to 7.

Let us try the procedure on Example 4.

Example 4
(6,31)

	Sum
You go first	
I select ___.	___
Your opponent selects 2.	___
I select ___.	___
Your opponent selects 1.	___
I select ___.	___
Your opponent selects 3.	___
I select ___.	___
Your opponent selects 1.	___
I select ___ and win.	___

Solution to Example 4 (6,31)

	Sum
You select 3.	3
Your opponent's selection is 2.	5
You select 5.	10
Your opponent's selection is 1.	11
You select 6.	17
Your opponent's selection is 3.	20
You select 4.	24
Your opponent's selection is 1.	25
You select 6 and win.	31

SG' TREATMENT MATERIALS

There is a procedure which will enable you to win any (6,31) game whenever you are allowed to make the first selection.

The appropriate first selection is determined by dividing 31 by 7. The remainder, 3 , should be your first selection.

Your later choices should be made so that your opponent's preceding choice plus your choice add up to 7.

Let us try the procedure on Example 4.

Example 4
(6,31)

	Sum
You go first.	
I select ___.	___
Your opponent selects 2.	___
I select ___.	___
Your opponent selects 1.	___
I select ___.	___
Your opponent selects 3.	___
I select ___.	___
Your opponent selects 1.	___
I select ___ and win.	___

Solution to Example 4
(6,31)

$$7 \overline{)31} \quad 4R3$$

	Sum
You select 3.	3
Your opponent's selection is 2.	5
You select 5.	10
Your opponent's selection is 1.	11
You select 6.	17
Your opponent's selection is 3.	20
You select 4.	24
Your opponent s selection is 1.	25
You select 6 and win.	31

G' TREATMENT MATERIALS

There is a procedure which will enable you to win any (6,31) game whenever you are allowed to make the first selection.

The appropriate first selection is determined by adding one to six, (1+6), and dividing 31 by this result. The remainder, 3 , of this division is the selection which should be made first.

After you have made the first selection your later choices should made so that your opponent's preceding choice plus your choice add up to 7. It is important to notice that 7=(6 + 1).

Let us try the procedure on Example 4.

Example 4
(6,31)

	Sum
You go first.	
I select ___.	___
Your opponent selects 2.	___
I select ___.	___
Your opponent selects 1.	___
I select ___.	___
Your opponent selects 3.	___
I select ___.	___
Your opponent selects 1.	___
I select ___ and win.	___

Solution to Example 4
(6,31)

$6+1 = 7$ $\qquad 7\overline{)31}$ R3

	Sum
You select 3.	3
Your opponent's selection is 2.	5
You select 5.	10
Your opponent's selection is 1.	11
You select 6.	17
Your opponent's selection is 3.	20
Your opponent's selection is 3.	20
You select 4.	24
Your opponent's selection is 1.	25
You select 6 and win.	31

Name _____
(print)

Test Booklet

You are going to be tested on your ability to play and win the type of game previously described. You will be playing against a person merely described as "your opponent." You will be allowed to make the first selection in each game. Blanks are left to indicate your selections. Also, blanks are left to indicate the sum. After your selection has been made and the sum computed and entered, then proceed to your opponent' s selection and calculate and enter his sum. The process is then continued until you win the game or your opponent wins or makes a selection which will make the running sum larger than the desired sum . <u>You must fill</u> in <u>all the blanks until the game is complete but there will probably be some extra blanks at the bottom of the page.</u> If you do not understand , raise your hand and a proctor will help you.

No erasures or markouts are allowed, so be sure of your entry before you mark it down. Now turn the page and proceed to test 1A.

Test 1A

The game is one which allows selections from the set {1,2,3,4,5,6} and the desired sum is 31.

	Sum
I select _____.	The sum is then _____.
Your opponent selects 5.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 6.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 4.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 1.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 3.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 4.	The sum is then _____.
I select _____.	The sum is then _____.

Turn the page to test 1B.

Test 1B

The game is one that allows selection from the set $\{1,2,3,4,5,6\}$ and where the desired sum is 31.

	Sum
I select _____.	The sum is then _____.
Your opponent selects 4.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 1.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 6.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 3.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 1.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 5.	The sum is then _____.
I select _____.	The sum is then _____.

Turn the page to test 2A.

Test 2A

The game is one which allows selection from the set $\{1,2,3,4,5,6\}$ and where the desired sum is 25.

	Sum
I select _____.	The sum is then _____.
Your opponent selects 6.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 4.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 1.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 5.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 2.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 6.	The sum is then _____.
I select _____.	The sum is then _____.

Turn the page to test 2B.

Test 2B

The game is one which allows selection from the set $\{1,2,3,4,5,6\}$ and where the desired sum is 29.

	Sum
I select _____.	The sum is then _____.
Your opponent selects 3.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 5.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 1.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 4.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 2.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 6.	The sum is then _____.
I select _____.	The sum is then _____.

Turn the page to test 3A.

Test 3A

The game is one which allows selection from the set $\{1,2,3,4,5\}$ and where the desired sum is 26.

	Sum
I select _____.	The sum is then _____.
Your opponent selects 3.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 4.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 4.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 1.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 2.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 5.	The sum is then _____.
I select _____.	The sum is then _____.

Turn the page to test 3B.

Test 3B

The game is one which allows selection from the set $\{1,2,3,4,5,6,7\}$ and where the desired sum is 33.

	Sum
I select _____.	The sum is then _____.
Your opponent selects 7.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 4.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 2.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 5.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 3.	The sum is then _____.
I select _____.	The sum is then _____.
Your opponent selects 6.	The sum is then _____.
I select _____.	The sum is then _____.